

ELECTRICAL ENGINEERING CIRCUIT CONCEPTS

Dr. Marcus O. Durham, PhD, PE

Dr. Robert A. Durham, PhD, PE

A Reference

The book is designed with three purposes:

A Text for introductory Electrical Engineering.

A Tutorial for other Engineering and science disciplines.

A Synopsis for review of fundamental concepts upon which all network analysis is based.

Chapter 1 - Electrical Fundamentals

Chapter 1 - Electrical Fundamentals	1
1.1 Introduction	4
1.2 Triad	4
1.3 Standard response	5
1.4 Domains	6
1.4.1 Time - t	6
1.4.2 Fixed - F	7
1.4.3 Laplace - s	7
1.4.4 Cyclic - $j\omega$	7
1.5 Physics	8
1.5.1 Space – the final frontier	8
1.5.2 Time – is underated	8
1.5.3 Matter – the elements of energy	9
1.5.4 Electromagnetic energy	9
1.5.5 Conservation	10
1.6 Electric measured	11
1.6.1 Voltage – potential	11
1.6.2 Current – flow rate	11
1.6.3 Time – frequency	12
1.7 Electric derived	12
1.7.1 Impedance	12
1.7.2 Power	13
1.7.3 Delay - angle	13
1.7.4 Electromagnetic energy redo	13
1.8 Sources	14
1.8.1 Voltage - current	15
1.8.2 Limits	15
1.8.3 Passive - active	15
1.9 Circuit - superposition	16
1.10 Electricity Bills	16
1.10.1 Residential electricity	17
1.10.2 Commercial Electricity bills	17
1.10.3 Average consumption	18
1.10.4 Phantom loads	18
1.11 Voltage ratings	19
1.11.1 Wiring systems	19
1.11.2 Nominal voltages	19
1.12 Units	20
1.12.1 Electrical units table	21
1.12.3 Manipulation	21
1.12.4 Length	22
1.13 References	23

1.1 Introduction

Electrical power costs are about one-third of industrial production operating costs. They represent a major part of most industrial companies' operating budget. Furthermore, electrical operations receive very little attention in proportion to their impact. Moreover, most operations are critically dependent on electrical energy. Whether for motors, computers or environmental systems electricity has become the most used and flexible energy form.

Electricity is a convenient form to transfer energy. Seldom is electrical energy used directly. Electrical systems always convert an available energy source to electrical energy. The electricity is then conveniently transferred to a load, which converts the electrical energy back to another energy form. The energy conversion on both ends generally goes through magnetics.

A generic electrical system covers equipment from a generator or power supply through controls to a motor or load.



The controls are generally a separate electrical system itself. In some problems, the system is analyzed directly. In small signal analysis, models are employed. In addition to technology, the design and installation of any electrical system must consider three major items - safety, environment, and cost.

Much of the physical system theory in this chapter is developed from a paper by the authors.

1.2 Triad

Electrical, as all physical systems, operates based on the Trinity or Triad Principle [1] which states:

Any item than can be uniquely identified can be further explained by three components.

A corollary states:

Two of the components are similar and project into one plane, while the third component is dissimilar and operates orthogonal.

The necessary terms for an electrical system can be identified using this grouping of three quantities. In a system, only 3 things are measured: voltage (V), current (I), and time (t).

Parameter	Symbol	Units	Measured	Description	Definition
Voltage	V	Volts	across	potential	$v_2 - v_1$
Current	I	Amps	through	flow rate	Coulomb/sec
Time	t	seconds	elapsed	duration	

From these variables, only three things can be calculated.

Parameter	Symbol	Units	Measured	Description	Definition
Impedance	Z	Ohms (Ω)	ratio	opposition	$\frac{V}{I}$
Power	S	Volt amps (VA)	product	Work over time	VI^*
Delay	td or \angle	seconds (degrees)	difference	Between V & I	$tV - tI$

1.3 Standard response

Because of the triad nature of physics, the results or response (Y) of any realizable circuit or system will fit a standard form. The response (Y) can be either voltage (V) or current (I).

A fundamental measurement of physical systems is time. Therefore, the response can be expressed in terms of time using the following standard form.

$$y(t) = F + (I - F)e^{-t/\tau} \cos(\omega t + \theta)$$

The response changes with time, $y(t)$. There are three components to the response - boundary conditions, transient, and cyclic.

Boundary conditions are the initial (I) and final (F) values. These are the values at the limits of time. At the instant of starting, the time is zero and the response has a beginning or initial value, I . At the end of consideration, the time approaches infinity and the response has an ending or final value, F . The final value is the direct current (dc) term. This value is fixed or constant.

Transient response changes with an exponential decay. The time constant (τ) is calculated from the impedance elements.

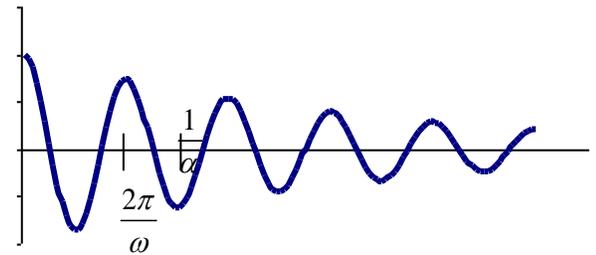
$$y(t) = e^{-\frac{t}{\tau}}$$

Cyclic response is a sinusoidal relationship. The frequency (ω) is in radians per second. Frequency (f) expresses how many cycles, revolutions, circles, or waveforms occur in a second. An angle results for the product of the frequency and time. A phase shift (θ) is an angle representing a delay from the fundamental reference frequency.

$$2\pi \text{ radians} = 360 \text{ degrees} = 1 \text{ cycle} = 1 \text{ revolution}$$

$$\omega = 2\pi f = \frac{\theta}{t}$$

Frequency is calculated from the elements of impedance. The cyclic or sinusoidal relationship is the alternating current (ac) term.



Common Values	
F	12 Vdc
I-F	169 Vpeak
f	60 Hz
ω	377 rad/sec
θ	30°
τ	1/10 sec

EXAMPLES

Ex 1.3-1	Given: $\tau = 1/2$ sec or $\tau = 1/10$ sec. Which goes to F the fastest? Solution: $\tau = 1/10$ sec is 5 times faster to reach the final value.
----------	---

Ex 1.3-2	Given: $f = 60 \text{ Hz}$, $\theta = 30^\circ$. Find: ω and time delay $\omega = 2\pi f = 2\pi \times 60 = 377 \text{ rad/s}$ $t_d = \frac{30^\circ}{360^\circ} \left \frac{\text{cycle}}{60 \text{ cycle}} \right \frac{\text{sec}}{720} = \frac{1}{720} \text{ sec}$
Ex 1.3-3	Given: A 60W lamp operating at 120 V. Find: Resistance $P = vi$ $= I^2 R$ $= \frac{V^2}{R}$
Ex 1.3-4	Given: $R = 10 \Omega$. Find: Impedance at dc, 60 Hz, 1 MHz. $Z = R = 10\Omega$ R is independent of frequency.

1.4 Domains

Suffice it to say, the response equation is a complex relationship. The mathematics to manipulate this relationship in combination with other similar equations would be complex indeed. Because of the uniqueness or complexity, calculations and manipulations in time are not always the easiest or best technique. For that reason a variety of analytical tools are used for investigations.

Lower case letters are used to represent instantaneous values and values that vary with time. Capitalization is used for representation in domains other than time.

The following discussion is technically accurate, but will not be developed using the normal mathematical rigor and procedures. Our objective is the understanding of concepts and use in a very specific and well-defined system, without the generalizations necessary for broader mathematical interpretation. Further details are covered in the math chapters.

1.4.1 Time - t

Time is a denominator function in most relationships. This gives a rate of change on the numerator. Sir Isaac Newton developed the Calculus as a tool that is specifically oriented to address this time rate of change. He multiplied the numerator and denominator by the rate of change operator (d).

$$\frac{d N}{d D}$$

For time varying systems the operation takes on a slightly different form that clearly relates the changing time or rate.

$$\frac{dN(t)}{dt}$$

That form has very specific and definite meaning, description, and definitions. The mathematical form is elegant, but the manipulation can be tedious. As a result the d/dt relationship is often transformed into other formats and domains.

1.4.2 Fixed - F

When the numerator is not changing, by definition, it is constant or fixed. This is by far the simplest relationship with time. This is the final value (F) of the standard response. Since there is no change, there is no decay. There is also no frequency.

$$f = \omega = 0$$

In the electrical world, this is called direct current (dc). It is a real value that provides the basis for conversion of electrical and magnetic energy into mechanical energy in the form of heat. Direct current energy is commonly used for small, portable, and transportation devices.

1.4.3 Laplace - s

The Laplace transform is an elegant tool that converts the d/dt into a simple algebraic multiplier. The definition of a Laplace depends on the transient response operating on the numerator function. The reciprocal of the time constant defines the domain operator, s .

The time differentiation property of the Laplace transform will be used as the basis for investigation.

$$\frac{dN(t)}{dt} = sN(s) - I$$

I is the initial condition. In a steady state, the initial condition is zero. Then the rate of time differentiation simply becomes a multiplication by the s operator.

$$\frac{d}{dt} = s$$

Standard tables are available that give other properties of the Laplace transform. From these properties, another table can be developed that shows the relationship between certain time signals and the resulting Laplace transform. Using transform pairs, direct substitutions can be made to move between the s -domain and the time-domain. This permits manipulation by the easiest to use form.

The s -operator becomes the value to solve for in the new equation. It is a complex number including the real term sigma (σ) and the orthogonal term frequency (ω).

$$s = \sigma + j\omega$$

Sigma (σ) is the inverse of the time constant and represents stability. Frequency (ω) is the number of cycles or revolutions in a second.

1.4.4 Cyclic - $j\omega$

The sinusoidal term of the response relationship has a consistent repetitive nature. When this is the only signal, the mathematics is greatly

$$e^{-st} = e^{-(\sigma + j\omega)t} = e^{-\sigma t} e^{-j\omega t}$$

$$e^{-\sigma t} = e^{-t/\tau}$$

$$e^{j\omega t} = \cos \omega t + j \sin \omega t$$

reduced. Under these sinusoidal steady state (SSS) conditions, then the Laplace transforms to the frequency domain.

$$s = j\omega$$

There is a direct correlation to the Fourier transform, which will be reserved for another discussion.

In addition to the frequency component, the cyclic term of the response relationship has an angle. The phase shift angle (θ) in represents a time delay between two signals operating at the same frequency.

In the electrical world this is called alternating current (ac). The most common signal for power devices is alternating current operating at 60 Hz in the western hemisphere and 50 Hz in Europe and other areas. As a result, ac or sinusoidal steady state analysis is a very common problem.

$$\omega = 2\pi f$$

$$= 2\pi \times 60 = 377 \text{ rad/s}$$

$$= 2\pi \times 50 = 314 \text{ rad/s}$$

1.5 Physics

The first part of this chapter was an introductory terminology of an electrical system. Before the system could be expanded, various mathematical perspectives had to be defined. Now the physics of the fundamental terms can be resolved.

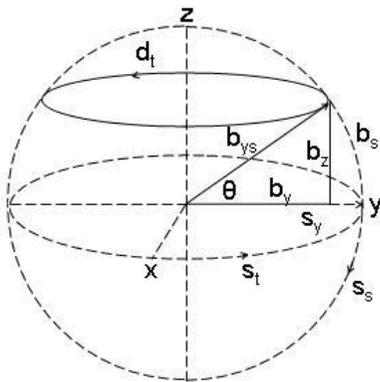
Physics is a very structured science based on observable or calculated events. There must be a starting point for analysis and descriptions.

In the beginning there was energy (W) or light.

In consideration of the Triad principle, the three ingredients of energy are space or distance, time, and matter.

1.5.1 Space – the final frontier

Space is typically a three-dimensional spheroid with measurements on the surface [1]. The most appropriate axes are longitude (s), latitude (t), and altitude or radius (r). In the figure, axes are represented by subscripts.



As would be expected, there are three types of distances. The first is the boundary volume, space, or size represented by 's'. Tangential or motion distance is represented by 'd'. The lever arm, torque, or working distance to the surface from the origin is expressed with the ray 'b_{ys}'. The ray is not along an axis, and the length may vary with the contour of the surface.

In a limited region, a rectangular (x,y,z) assumption is acceptable and is used for most linear analysis. Vectors relative to other axes are projected onto the rectangular axes.

1.5.2 Time – is underated

As would be expected from the triad principle, in the complete physical model, there are three different time - based components. In some equations time is constant, so $t=1$. The second time, t_v , is associated with velocity or rate and energy. The third time, t_a , is associated with acceleration, potential, and power. The times are independent of each other in duration and direction, but in short intervals they may be concurrent.

Time can take on the three realizations - as a fixed or constant, as rotational, cyclic or clock (t_r), and as linear or calendar (t_c). As an illustration, the 24 hours in 1 day are concurrent with a day in a calendar. However, the next day cycles again, but the calendar moves forward. In the short intervals used for differential calculations, the times usually are concurrent.

When both variable times are used in an equation, it is called second order.

In all the fundamental concepts, time is a denominator value. The ratio of a concept to time gives a rate. A change with respect to time causes an angular shift of 90° or $\pi/2$. The second time causes another 90° shift. The result of a the second shift is a direction opposite to the original orientation.

$$\frac{d}{dt} = s = j\omega$$

1.5.3 Matter – the elements of energy

The three forms of energy are derived from the three elements of matter - mass (m), charge (q), and magnetic pole strength (p). Mass yields mechanical energy, charge yields electric energy, and magnetic poles yield magnetic energy.

Mass is measured in grams. Charge is measured in Coulombs. Magnetic pole strength is measured in Webers.

Matter	Sym	Energy	Unit	Abb
mass	m	mechanical	gram	g
charge	q	electrical	Coulomb	C
pole strength	p	magnet	Weber	Wb

1.5.4 Electromagnetic energy

Durham's electromagnetic energy law affirms the change in the product of charge and pole strength over time [1]. That represents two-thirds of matter.

$$W = \frac{pq}{t}$$

The relationship is the point or node form of electromagnetic energy. The field form is based on the motion or operating volume (V_D) over the boundary volume (V_S).

$$\begin{aligned} W &= \frac{pq}{t} \frac{V_D}{V_S} \\ &= \frac{pq}{t} \frac{b_{ys} d_T s_Y}{s_S s_T s_Y} \end{aligned}$$

The energy density (Joules/m³) is the electric intensity times the density. It is also the magnetic intensity times the density.

$$\text{For } b_{ys} = 2m \text{ \& } \theta = \pi/6$$

$$b_s = 2 \times \pi/6 = \pi/3$$

$$b_z = 2 \times \sin \pi/6 = 1$$

$$b_y = 2 \times \cos \pi/6 = 1.732$$

$$s_y - b_y = 2 - 1.732 = 0.268$$

$$d_T = 2\pi \times b_y = 10.88$$

accel + vel + pos

$$\frac{x}{t^2} + \frac{x}{t} + \frac{x}{1}$$

$$\frac{d^2x}{dt^2} + \frac{dx}{dt} + x$$

$$xs^2 + xs + x$$

$$1 \text{ C} = 6.241506 \times 10^{18} \text{ electrons}$$

$$1 \text{ electron} = -1.6022 \times 10^{-19} \text{ C}$$

$$\text{Joule} = \frac{\text{Weber} \cdot \text{Coulomb}}{\text{sec}}$$

$$\begin{aligned} \mathcal{E} &= \left[\frac{p_z}{t_r s_t} \right]_t \\ \mathcal{H} &= \left[\frac{q_y}{t_r s_s} \right]_s \\ \mathcal{D} &= \left[\frac{q_y}{A_y} \right]_y \\ \mathcal{B} &= \left[\frac{p_z}{A_z} \right]_z \end{aligned}$$

The electric intensity ' \mathcal{E} ' is voltage over the measurement path, ' s_t '. The magnetic intensity ' \mathcal{H} ' is current through a closed path, s_s . Note that the current path encloses the voltage path. The electric density, \mathcal{D} , is the charge over the area. The magnetic density, \mathcal{B} , is the flux over the area. Note that the areas are perpendicular.

The density form of electromagnetic energy is obtained by reducing the directional vector (s_y) from both the numerator and denominator.

$$W = \frac{pq}{t} \frac{bd}{s_S s_T}$$

Obviously there is a very close relationship between charge and magnetics. One does not depend on the other. They can exist independently as seen in the node form. However, when there is motion represented by the rate in the field form, then one influences the other. The motion of charge and magnetics is the basis of electric machines and electromagnetic fields.

1.5.5 Conservation

All real, physical systems adhere to the law of Conservation of Energy.

There is nothing new under the sun.

or, more traditionally,

The sum of the energy in a closed system is zero.

$$\Sigma W = 0$$

Energy is neither created nor destroyed, it may only change form.

When applying the conservation of energy relationship to the node form of the electromagnetic energy equation, two new conservation laws necessarily develop. These are conservation of charge and conservation of magnetics.

Unfortunately energy conversion is not very efficient. The energy conversion process is typically about 25-45%. The remainder goes into heat. A heat recovery system can be used to recoup some of the heat and convert it into additional electricity.

A typical small gas turbine uses 14,000 BTU/kWhr or roughly 25% efficient. An aircraft derivative turbine is about 45% efficient, while a combined-cycle system is approximately 55%.

Energy

1000 BTU=
1 cf gas=
0.239 kWh=
1,055,056 J

EXAMPLES

Ex 1.5-1 Given: 100,000 protons
Solution: $10^5 \times 1.6022 \times 10^{-19} C = 1.6 \times 10^{-14} C = 0.016 pC$

Ex 1.5-2 Given: 10 Watts at a magnetic flux of 5 Webers. Find the charge.

$$W = \frac{pq}{t}$$

$$q = \frac{W \times t}{p} = \frac{10}{5} = 2 \text{Coulomb}$$

Ex 1.5-3 Given: A small generator uses 100 MJ of natural gas and delivers 10 kWh of electricity. What is the difference in energy and where does it go?

$$W_{in} - W_{out} = W_{heat}$$

$$100 \times 10^6 J - \left[\frac{10 \times 10^3 \text{Wh}}{\text{hr}} \left| \frac{3600 \text{sec}}{\text{hr}} \right| \frac{J}{\text{W} \cdot \text{sec}} \right] = 64 \text{MJ}$$

Ex	Given: $R = 10 \Omega$.
1.5-4	Find: Impedance at dc, 60 Hz, 1 MHz. $Z = R = 10\Omega$ R is independent of frequency.

1.6 Electric measured

As introduced earlier, the three electrical parameters that can be measured are voltage, current, and time. With the physics and mathematical response information, these can be defined. The definitions are contained in the electromagnetic energy equation.

1.6.1 Voltage – potential

Voltage (V) - measured as volts - is the potential force or pressure in a circuit. It exists whether anything is connected or not. Voltage is the analog of pounds per square inch in a hydraulic system.

From Durham's electromagnetic energy law, the definition of voltage is the change in the magnetic flux over time, when charge is constant or not changing.

$$V = \frac{p}{t} \Big|_{q=k}$$

$$V = \frac{dp}{dt} = sp$$

Magnetic poles always exist in pairs. However, the poles strength or flux will vary between the poles.

Voltage is measured across or as the difference between two points. The positive connection is represented by a plus (+) sign and is the first letter in the subscript. The negative connection is represented by a minus (-) sign and is the second letter in the subscript sequence.

$$V_{ab} = V_a - V_b = -V_{ba}$$

A voltage source has current flowing from the positive terminal. A voltage drop arises from current flowing into the positive terminal of a load. When it is a source, voltage is also called *electromotive force (emf)*. As a result the symbol E may be used for voltage.

1.6.2 Current – flow rate

Current (I) - measured as amps - is the rate or quantity of flow through a path. Current can be measured only if a load is connected and operating. Current is the analog of gallons per minute in a hydraulic system.

From Durham's electromagnetic energy law, current is the change in the charge over time when magnetics is constant or not changing.

$$I = \frac{q}{t} \Big|_{p=k}$$

$$I = \frac{dq}{dt} = sq$$

In physical systems charge is always an integral multiple of the charge on a single electron. Current is measured through a path. Positive current flows from the positive terminal of a voltage source and returns at the negative terminal. When it is a driving source, current is also called *magnetomotive force (mmf)*.

1.6.3 Time – frequency

Time (t) - measured in seconds - is the difference in time between events. The reciprocal of time is the frequency.

From Durham's electromagnetic energy law, frequency is one over time.

$$f = \frac{1}{t} \Big|_{pq=k}$$

Time and frequency are integral components of all electrical analysis.

Spectrum	Frequency
Direct current	0
AC power	60 Hz
Sound	1 kHz
AM radio	1MHz
FM radio	100 MHz
UHF TV	500 MHz
Cell phone 1	800 MHz
Cell phone 2	1.9 GHz
Satellite radio	2.2 GHz
Wireless LAN	2.4 GHz
Microwave	2.5 GHz
Radar	5.0 GHz
Infrared	
Visible	
Ultraviolet	
X-ray	
Gamma ray	
All objects have an electro-magnetic (radio) frequency.	

EXAMPLES

Ex 1.6-1 Given: A magnet moves in a motor causing a flux change of 30 Wb in 0.25 sec.

$$V = \frac{dp}{dt} = \frac{30Wb}{0.25sec} = 120V$$

Ex 1.6-2 Given: $q = 10 \cos 377t$
Find I at $t = 0.5$ sec.

$$I = \frac{dq}{dt} = -10 \sin(377t)$$

$$= -10 \sin(377 \times 0.5) = -0.044A$$

1.7 Electric derived

All electrical relationships can be calculated from the two terms - voltage and current - in conjunction with time.

1.7.1 Impedance

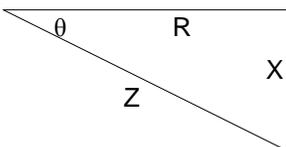
Impedance (Z) - measured in Ohms - is the ratio of voltage to current. Impedance is the opposition to current flow. The relationship is called Ohm's Law.

$$Z = \frac{V}{I}$$

Impedance is a complex parameter that is a characteristic of how electrical conductors are arranged.

$$Z = R + jX$$

R is resistance and X is called reactance.



1.7.2 Power

Power (S) - measured in volt-amps - is the product of voltage and current.

$$S = VI^*$$

Power is a complex parameter also called apparent power. The terms and angle correlate directly to the impedance

$$S = P + jQ$$

The real component (P) converts electrical to mechanical energy and is measured in watts. Power is energy or work (W) that occurs in some period of time.

$$P = \frac{dW}{dt} = sW$$

The imaginary power Q has a component for electrical energy and one for magnetic energy. It is measured in VoltAmpReactive.

The instantaneous real power is the product of the instantaneous voltage and current in time.

$$p = vi$$

Using the impedance relationships, power computations can be expanded.

$$\begin{aligned} S &= VI^* \\ &= I^2 Z \\ &= \frac{V^2}{Z} \end{aligned}$$

1.7.3 Delay - angle

Voltage has a magnitude and angle. The angle translates to a time determined from the operating frequency. Similarly, current has a different angle and associated time. The calculations for impedance and power have a resulting angle (θ) which represents the time delay between the voltage and current angles.

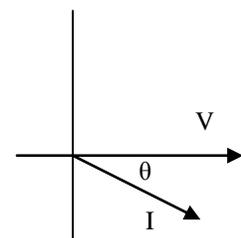
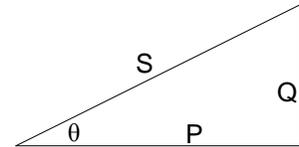
$$\theta = \angle V - \angle I = \angle Z = \angle S = \cos^{-1} pf = 2\pi ft$$

$$t_d = t_v - t_i$$

The time delay is also called a phase shift. *Power Factor (PF)* is the time block between voltage and current expressed in angular terms. It is the phase shift between voltage being at a maximum and current being at a maximum.

1.7.4 Electromagnetic energy redo

The electromagnetic energy can now be expressed in another set of terms. These are from the definitions of voltage and current. In most situations, the voltage or current relationship is used. The average energy stored by the impedance components will be investigated in the next chapter.



$$W = \frac{pq}{t} = Vq = Ip$$

EXAMPLES

Ex 1.7-1	<p>Given:</p> <p>Voltage = 120V</p> <p>Current = $15\angle -30^\circ$ A</p> <p>Time = 2hours</p>
Ex 1.7-3	<p>Solution:</p> $Z = \frac{V}{I} = \frac{120}{15\angle -30^\circ} = 8\angle +30^\circ \Omega$ $R = Z \cos \theta = 8 \cos 30^\circ = 6.9\Omega$ $X = Z \sin \theta = 8 \sin 30^\circ = 4\Omega$
Ex 1.7-4	<p>Solution:</p> $S = VI^* = (120)(15\angle +30^\circ) = 1800\angle +30^\circ \text{ VA}$ $P = S \cos \theta = 1800 \cos 30^\circ = 1559W$ $Q = S \sin \theta = 1800 \sin 30^\circ = 900\text{VAR}$
Ex 1.7-5	<p>Solution:</p> $P = \frac{dW}{dt}$ $W = Pt = (1559W)(2hr) = 3118Wh = 3.118kWh$ $W = \frac{3118Wh}{1} \left \frac{60 \text{ min}}{\text{hr}} \right \left \frac{60 \text{ sec}}{\text{min}} \right \left \frac{J}{W \text{ sec}} \right = 11.2 \text{MJoule}$
Ex 1.7-5	<p>Solution:</p> $t_d = \frac{30^\circ}{360^\circ} \left \frac{\text{cycle}}{60 \text{ cycle}} \right \left \frac{\text{sec}}{720} \right = \frac{1}{720} \text{sec}$ $\text{pf} = \cos \theta = \cos 30^\circ = 0.866$
Ex 1.7-6	<p>Given: $V = 1 \sin \theta$ and $I = 1 \cos \theta$</p> <p>Find: power at 0, 30, 45, and 90 degrees</p> $p = (\sin 0^\circ)(\cos 0^\circ) = 0W$ $= (\sin 30^\circ)(\cos 30^\circ) = 0.433W$ $= (\sin 45^\circ)(\cos 45^\circ) = 0.5W$ $= (\sin 90^\circ)(\cos 90^\circ) = 0W$

1.8 Sources

Sources provide the drive for a system. There can be either voltage or current sources. The source is the limit on voltage or current. Normal sources are either direct current (dc) or alternating current (ac).

1.8.1 Voltage - current

Independent sources provide rated energy without outside influence. Independent sources are represented by a circle. Dependent sources provide energy in response to an external voltage or current. Dependent sources are represented by a diamond. The dependent source may be a voltage or current and it may be controlled by voltage or current. That would make four possible combinations for dependent sources.

A voltage source maintains $v=V_o$ for any current I flowing from the source. Current sources maintains $i=I_o$ for any voltage V across the source.

Source	Symbol
DC or constant voltage sources	
AC sources $v = V_o \cos \omega t$	
Independent current source	
Dependent current source	
Dependent voltage source	

1.8.2 Limits

There are is a lower and upper limit on impedance. A short circuit is near zero impedance. An open circuit is near infinite impedance. A short circuit has no voltage drop. An open circuit has no current flow.

A short circuit is defined as $v=0$ for any i	
An open circuit is defined as $i=0$ for any v	

1.8.3 Passive - active

Current flow is indicated by an arrow. Positive current is in the direction of the arrow. This is the opposite direction of electron movement. Current leaves a positive source and enters the negative side of the source. A positive voltage source causes a rise in voltage.

The circuit is completed by flow through opposition called impedance. A voltage drop occurs when current enters the positive terminal of impedance.

Power is received or passive if the direction of current flow is from plus to minus terminal. Power is delivered or active if the current flows from minus to plus.

EXAMPLES		
Ex 1.8-1	<p>Given: The AC voltage source is 120 V. Find: Potential between the Hot & Neutral terminals on the receptacle. Solution: $V_{in}=V_{out}$. The voltage is 120 V regardless of the current flow.</p>	
Ex 1.8-1	<p>Given: The current source provides 10A. Find: Current into the top load. Solution: $I = 10 - 3 = 7A$</p>	

1.9 Circuit - superposition

A *circuit* is a combination of sources and impedances. Superposition allows the separation of a complex problem and circuits into smaller parts. Superposition means the weighted sum of the forcing functions causes a weighted sum of the responses due to each forcing function acting alone. In other words, the circuit can be solved with one source at a time, then the results are combined.

1.10 Electricity Bills

Electrical utility bills are based on three components – fixed costs, energy costs, and demand costs. Fixed costs are to recover the handling costs of the bill. The fixed cost is relatively small compared to the total bill.

Demand charges are based on the total connected power in kilowatts. The amount is typically based on a peak value within the season. The cost is to recover the investment for the equipment installed.

Energy charges are the variable costs based on the amount of energy necessary to operate the generators. The units are kilowatt-hours. The cost of fuel varies based on quantity, season, availability, competition from suppliers, and requirements by other users. The utility bill generally has a set amount for the kilowatt-hours plus an adjustment factor for the fuel variable costs.

There are often other adjustments to the bill. For example there may be a penalty for power factor less than 95%. There may be credits for equipment, such as transformers, that are owned by the customer.

Generally, residential utility bills have only a fixed cost and energy charge with fuel cost adjustments. Larger consumers will have demand charges and other adjustments. The demand cost is included in the energy charge for residential. In addition, economies of scale reduce the investment to supply a large quantity of power. Therefore, the cost of electricity costs less per kilowatt-hour for large users than for small.

Because electricity is a regulated commodity, the political pressures often force rate schedules that do not accurately reflect the true cost of the electricity. For example, the regulatory authority may dictate that very small usage for senior citizens be less than costs. Obviously, someone else will have to pay more to make up the difference.

$$S = VI^*$$

$$Z = \frac{V}{I}$$

$$S(j\omega) = P + j(Q_L - Q_C)$$

$$Z(j\omega) = R + j(X_L - X_C)$$

\$
universal
engineering
symbol

\$, t, quality
engineering trade-offs

1.10.1 Residential electricity

A typical residential rate schedule may have the following form. Since costs are so dynamic and vary for each utility and region of the country, this is only representative. All costs in the illustration are amounts / month

Schedule	kWh	\$/kwh	Fixed
Base cost			\$12.00
First	100	0.16	
Next	200	0.10	
Over	300	0.06	

Obviously this is readily structured for use with a spreadsheet.

1.10.2 Commercial Electricity bills

Commercial bills have similar schedules to residential plus they have a demand schedule. Since the fixed cost for investment is separated out, the energy cost is typically less.

Schedule	kWh	\$/kwh	kW	\$/kW	Fixed \$
Base cost					100.00
First	100,000	0.05			
Next	200,000	0.045			
Over	300,000	0.04			
First			100		1000.00
Next			100	10.00	
Over			200	8.00	

EXAMPLES

Ex 1.7-1 Given: Residential user consumes 700 kWh in one month. Compute the electric bill.

Schedule	kWh	\$/kwh		Fixed \$	Qty used	Amount \$
Base cost				12.00		12.00
First	100	0.16			100	16.00
Next	200	0.10			200	20.00
Over	300	0.06			400	24.00
Total					700	72.00

The average cost for the energy is $\$72/700 = \$0.102 / \text{kWh}$.

Ex 1.7-1 Given: A commercial user consumes 350,000 kWh with a 500 kW demand. Compute the electric bill.

Schedule	kWh	\$/kwh	kW	\$/kW	Fixed \$	Qty used	Amount \$
Base cost					100.00		100.00
First	100,000	0.06				100,000	6,000.00
Next	200,000	0.05				200,00	10,000.00
Over	300,000	0.04				50,000	2,000.00
First			100		1000.00		1000.00
Next			100	10.00		100	1000.00
Over			200	9.00		300	2700.00
Total							22,800.00

The average costs per kilowatt-hour is $\$22,800/350,000 = \0.065 . Although this is lower than the residential rate, it reflects the improved economics of cost of service on a large scale.

1.10.3 Average consumption

The typical consumption for common devices is shown in the table. Similar devices will have comparable usage.

Appliance	Watts	Estimated cost
Blow-dryer	1200	2¢/15 min.
CD Player	10	1¢/15 hrs
Central Vacuum	1440	10¢/hr
Clothes Dryer	5500	27¢/load
Computer	450	3¢/hr
Deep Fryer	1448	11¢/hr
Dishwasher	1201	7¢/load
Electric Blanket	177	1¢/hr
Freezer - Manual Defrost 20 cu.ft.	600	32¢/day
Freezer - Frostless 20 cu.ft.	790	43¢/day
Heater - portable	1500	11¢/hr
Microwave Oven	1450	11¢/hr
Mixer	127	1¢/hr
Range w/oven	12000	22¢/day
Range w/Self-Cleaning Oven	13700	23¢/day
Refrig/Freezer - Frostless 20 cu.ft.	780	40¢/day
Television	110	4¢/day
Vacuum Cleaner	1300	9¢/hr
Washing Machine	512	2¢/load
Water Heater	4500	94.5¢/day

1.10.4 Phantom loads

Phantom loads are the small amount of energy required to keep most electronic devices powered, even when they are not being used. On an individual basis these are not large. Together they run 50 – 100 kWh per month in the average home. The summation of that load within one utility or within the country begins to represent a significant number.

Device	Phantom W
Microwave	2-6
Answering Machine	2-3
Cordless Phone	2-4
CD Player	3-8
Television	0-12
VCR	1-15
Computer	0-2
Surge Suppressor	0.2-0.4
Oven Clock	3-4
Security System	6-22
Cable Box	8-15
Battery Charger	2-5

1.11 Voltage ratings

1.13.1 Wiring systems

All common electrical power is carried in conductors or wires. The arrangement of these wires determines their function.

Direct current uses a red wire for positive and a black wire for the negative.

Single-phase (1 \emptyset) is an electrical system that uses only two current carrying conductors. The hot or positive side is black and the neutral or common is white.

Three-phase (3 \emptyset) is a system that uses three current carrying conductors. Any color can be used although black is the most common.

A *ground* is used as a reference. It is also for safety purposes. A *grounding* wire may be present in any system. The wire will have green insulation or will be bare.

A *neutral* is a carrying wire that is also *grounded*. It may be present in either the single phase or three phase system. It is the common for a single-phase system. It is the fourth wire of a three-phase system. The insulation is white.

1.11.2 Nominal voltages

General: There are many different system voltage levels. Some of the common ones are listed. Others are in use at various locations.

Controls: Controls are often less than 50 volts for safety considerations. Voltages less than this usually can be contacted without fatal consequences. The most common systems employ 48, 24, 12, 6, and 5 volts. Nevertheless, some systems safely retain 120 volts for convenience.

<48 120

Secondary (Utilization): Most power equipment operates at these levels. The first number represents the voltage between a line and ground, while the second number represents the voltage between two lines. The line-to-line voltage is the number used for nominal system voltage rating on three-phase systems.

2400 / 4160 277 / 480 240 120 / 208

Typical applications fit in the matrix. System requirements may dictate other combinations.

Volts	Phase	Class	Size
4160	3	extra large	>1000 Hp
2400	3	very large	>250 Hp
480	3	large	>3 Hp
277	1	lighting	commercial
240	1	general	>1 Hp
208	3	motors	>1 Hp
120	1	general	<1 Hp

Primary (Distribution): For distribution voltages, typically one suspension insulator bell corresponds to approximately 10,000 volts.

2400 / 4160 7200 / 12470 7620 / 13200
7970 / 13800 14400 / 24940 19920 / 34500

Transmission: For transmission voltages, typically one suspension insulator bell corresponds to approximately 20,000 volts.

34500 69 KV 138 KV 240 KV

Extra High Voltage: There are only a limited number of these systems. Cost and concerns about hazards have limited their acceptance.

345 KV 700 KV 1 MV >500 KV DC

1.12 Units

Behind the Math

The US system is based the English Customary Weights and Measures system. This system has its basis in easily measurable units that were well known to residents in an agricultural society. For example, an *inch* derived from the width of a thumb, a *yard* came from the distance between the tip of one's nose and the tips of the fingers on an outstretched arm, and a *gallon* was the size of a container that would carry 8 pounds of wheat. Today all US customary units are clearly defined, and governed by the National Institute of Standards and Technology (NIST).

The US Customary Units system is the historical system of measurement in the United States. This system is comprised of units such as foot, pound, gallon, yard, teaspoon and mile. It is, mathematically, more difficult to work with than the metric system, but it is still firmly entrenched in the US culture.

The Metric Unit system is commonly referred to as the SI system. This stands for the *Système International d'unités*, or International system of units. The SI system is based on seven base units: one each for length (meter), mass (gram), time (second), electric current (Ampere), temperature (Kelvin), amount of substance (mole), and luminous intensity (candela). From these base units, the SI system uses several prefixes, all based on powers of

Multiplier	Prefix Name	Prefix Symbol
10^{24}	yotta	Y
10^{21}	zetta	Z
10^{18}	exa	E
10^{15}	peta	P
10^{12}	tera	T
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^2	hecto	h
10^1	deca	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a
10^{-21}	zepto	z
10^{-24}	yocto	y

10, to further describe the multiples of these base units. A list of these prefixes and multipliers is shown in the table to the right, along with the common symbols.

Application of these prefixes is very straightforward. A kilo-meter (km) is 10³ meters, or 1,000 meters. A nano-second (nsecond) is 10⁻⁹ seconds, or $\frac{1}{1,000,000,000}$ of a second. A tera-gram (Tgram) is 10¹² grams, or 1,000,000,000,000 grams.

1.12.1 Electrical units table

The table lists the significant units used in electrical systems.

<i>Quantity</i>	<i>Symbol</i>	<i>Units</i>	<i>Abbrev</i>
Voltage	V	Volt	V
Current	I	Ampere	A
Time	t	second	s
Frequency	f	Hertz	Hz
Impedance	Z	Ohm	Ω
Resistance	R	Ohm	Ω
Capacitance	C	Farad	F
Inductance	L	Henry	H
Conductance	G	Siemens	S
Apparent power	S	Volt-ampere	VA
Reactive power	Q	VA _r	var
Real Power	P	Watt	W
Energy	W	Watt-hour	Wh
Energy, work, & heat	W or E	Joule	J
Battery charge	q	ampere-hour	Ah
Mass	m	gram	g
Electric charge	q	Coulomb	C
Magnetic pole strength	p	Weber	Wb
Magnetomotive force	mmf	Ampere-turn	A-t
Magnetic field strength	<i>H</i>	Ampere per meter	A/m
Magnetic flux density	<i>B</i>	Tesla	T
Electric field strength	<i>E</i>	Volt per meter	V/m
Electric flux density	<i>D</i>	Weber pe	Wb/m ²
Force	F	newton	N
Celsius temperature		degree Celsius	°C
Thermal conductivity		watt per meter kelvin	W/(m-K)

By convention of the IEEE, any electrical unit named for an individual is capitalized. However, scale modifiers such as kilo are not capitalized. Abbreviations follow the capitalization of the name. As a result abbreviations can look unusual. For example, one thousand Volts is a kilovolt, of kV.

The standard abbreviations for scaling values in powers of 10 are shown in the tables.

1.12.3 Manipulation

Numerous equations and relationships are used in engineering calculations. These are the basis for most electrical, magnetic, and mechanical machine analysis. The study requires manipulation of relationships in a variety of ways and combinations.

There is a simple technique for systematic manipulation: *Begin with a known quantity. Draw a horizontal line so the units can be manipulated until the correct form is obtained. Then the equation will be correct, except for conversion constants.*

EXAMPLE

Ex 1.11-1 Given: Convert revolutions per minute (RPM) to radians per second.

$$\frac{\text{rev}}{\text{min}} \left| \frac{\text{min}}{60\text{sec}} \right| \frac{2\pi \text{ rad}}{\text{rev}} = \frac{\pi}{30} \text{ rad/sec}$$

1.12.4 Length

The US system of length has many different units. The base unit is the inch, with all other units derived in some way from it. There are many different measurements of length in the US system including the foot, yard, and mile.

The metric system of measurement is based on the meter. Historically, a meter was defined as $\frac{1}{10,000,000}$ of the distance from the North Pole to the South Pole, when measured through Paris. What scientists found, however, was this measurement was not easily repeatable. The meter is now defined as the length of the path that light will travel, in a vacuum, in $\frac{1}{299,792,458}$ of a second.

The table below contains common length measurements, and their relationship to each other.

Measurement	US Units	Metric Units
1 Angstrom (Ang)	100,000 Fermi	1.1 nanometers(nm) $1.1 * 10^{-9} \text{ m}$
1 mil	254,000 Ang 0.001 in	0.254 micrometers(μm) $2.54 * 10^{-5} \text{ m}$
1 millimeter (mm)	39.37 mils 0.0394 inch	1.0 millimeter (mm) 0.001 m
1 centimeter (cm)	0.3937 in	10 millimeters (mm) 0.01 m
1 inch (in)	1,000 mils	2.54 centimeters (cm) 0.0254 m
1 foot (ft)	12 in	30.48 centimeters (cm) 0.3048 m
1 yard (yd)	3 ft 36 in	0.9144 m
1 meter (m)	3.281 ft 39.37 in	1.0 m
1 fathom (fath)	6 ft 72 in	1.8288 m
1 rod (rd)	5.5 yd 16.5 ft	5.0292 m
1 furlong (fur)	40 rd 660 ft	0.201168 km 201.168 m
1 kilometer (km)	1093.6 yd 3280.8 ft	1.0 km 1000 m
1 mile	1760 yd 5280 ft	1.609 km 1,609 m

The conversion factors in the tables can be rewritten in a fraction form as shown below.

$$\frac{1yd}{3ft} \quad \text{or} \quad \frac{3ft}{1yd}$$

$$\frac{1m}{39.37in} \quad \text{or} \quad \frac{39.37in}{1m}$$

To choose which conversion factor to use, select one that has the units desired in the answer in the numerator, and the units given with the measurement in the denominator. When you have the same units in the numerator and denominator of an equation, the units will cancel each other out.

1.13 References

[1] "A Composite Approach to Electrical Engineering," Marcus O. Durham, *Institute of Electrical and Electronic Engineers Region V*, 88CH25617-6/000-143, Colorado Springs, CO, March 1988, pp 143-147.

Applications Engineering Approach to Maxwell and Other Mathematically Intense Problems", Marcus O. Durham, Robert A. Durham, and Karen D. Durham, *Institute of Electrical and Electronics Engineers PCIC*, September 2002.

"Applications Engineers Don't Do Hairy Math", Marcus O. Durham, Robert A. Durham, and Karen D. Durham, *Proceedings of 35th Annual Frontiers in Power Conference*, OSU, Stillwater, OK, October 2002.

"Electromagnetics in One Equation Without Maxwell", Marcus O. Durham, *American Association for Advancement of Science - SWARM*, Tulsa, OK, April 2003.

