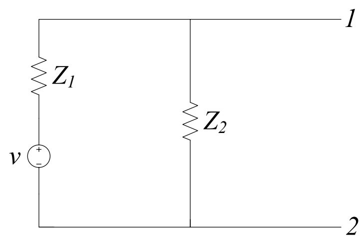
Chapter 5 – Thevenin

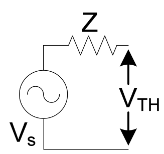
5.1 Introduction



Thevenin and Norton equivalent circuits are used to simplify any network to an equivalent impedance and a single source (driver). These are the values of a two terminal circuit that could replace the entire network. Yet the equivalent network will produce the same effect at the terminals of a load or incident impedance under investigation. The terminals will be across a load or incident impedance.

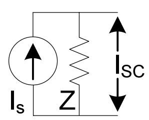
5.2 Thevenin Norton Equivalent

A Thevenin equivalent circuit consists of a voltage source in series with an equivalent impedance, Z_{EQ}. The voltage is the open circuit voltage, V_{OC}, across the terminals.



A Norton equivalent circuit consists of a current source in parallel or shunt with the equivalent impedance. The current is the short circuit, I_{SC}, through the terminals. It is the current that shorts the load impedance.

The equivalent impedance is the combination of all other impedances except the load. The equivalent impedance is measured at the terminals across the load. The equivalent impedance is the same for both networks.



If either the Norton or Thevenin equivalent is known, the other can be found from Ohm's Law.

$$Z_{EQ} = \frac{V_{OC}}{I_{SC}} = \frac{V_{TH}}{I_{SC}}$$

$$S = VI^*$$

$$Z = \frac{V}{I}$$

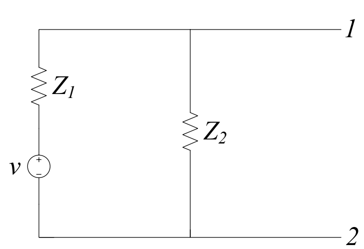
The following rules help in finding the Thevenin or Norton equivalent circuits. The equivalent network is calculated by taking the sources to the limit. The impedance L is manipulated the same as R. The impedance C in series is manipulated like R in parallel.

$$S(j\omega) = P + j(Q_L - Q_C)$$

$$Z(j\omega) = R + j(X_L - X_C)$$

5.3 Equivalent impedance

The equivalent impedance is measured from the two terminals of the incident impedance. This will produce the same load as the network. The equivalent impedance is a value that is determined by taking the source to the limit and finding the series / parallel impedance.



1) Select the terminals of the incident impedance that will be supplied by the equivalent circuit.
2) Replace independent voltage (V) sources by its internal impedance. It approaches a short, where V=0.
3) Replace independent I sources by its internal impedance. It approaches an open, where I=0.
4) Calculate the impedance. Use series / parallel rules.
5) If the voltage supplied at the node and the current into the node are known, then $Z = \frac{V}{I}$.

5.3.1 Illustration

Find the equivalent impedance at the terminals of the network in the figure.

Replace the voltage source with a short.

Calculate the remaining impedance.

$$Z_{EQ} = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

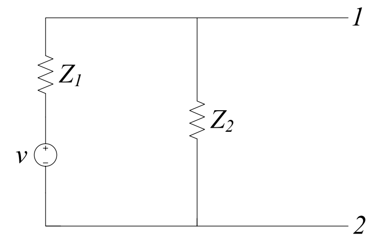
5.4 Thevenin voltage

The Thevenin voltage is the open circuit voltage at the terminals of the two-port network. The Thevenin voltage is the forcing function that will produce the same voltage at the terminals of the incident impedance.

1) Select the terminals of the incident impedance that will be supplied by the equivalent circuit.
2) Open the circuit terminals across the incident impedance.
3) Leave the sources active.
4) Calculate the voltage across the open terminal. This will be the voltage across the incident impedance.

5.4.1 Illustration

Find the equivalent open-circuit voltage at the terminals of the network in the figure.



Open terminals and calculate v.

V_{TH} = voltage across Z_2.

This is a series circuit with voltage divider.

$$V_{TH} = V_S \left(\frac{Z_2}{Z_1 + Z_2} \right)$$

Alternatively, the current through impedance R_2 could be found. Then the voltage across the impedance could be calculated.

$$I = \frac{V_S}{Z_1 + Z_2}$$

$$V_{TH} = IZ_2 = \frac{V_S Z_2}{Z_1 + Z_2}$$

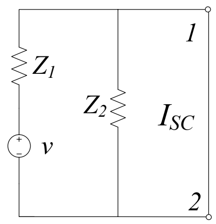
5.5 Norton current

The Norton current is the short circuit current that would bypass the terminals of the incident impedance two-port network.

1) Select the terminals of the incident impedance that will be supplied by the equivalent circuit.
2) Short circuit terminals of the two-port incident impedance.
3) Leave the sources active.

4) Calculate the current through the shorted terminal. This will be the current that shorts (bypasses) the incident impedance.

5.5.1 Illustration



Find the equivalent short-circuit current at the terminals of the network in the figure.

Short the terminals and calculate the current. I_{sc}=I through terminals.

Impedance R_2 is bypassed. Compared to the short, $R_2 \rightarrow \infty$.

The current is found from the remaining series circuit. I_{sc}= I in series circuit.

$$I_{SC} = I = \frac{V_S}{Z_1}$$

5.6 Check-up

The open-circuit voltage is the Thevenin voltage. The short-circuit current is the Norton current. They are related by the equivalent impedance

$$Z_{EQ} = \frac{V_{TH}}{I_{SC}}$$

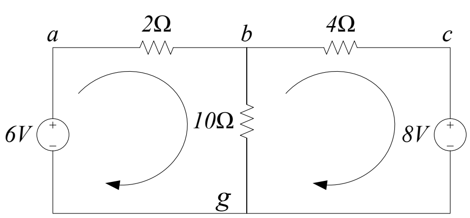
This provides a check for the calculations. Substitute the values for the voltage and current in the impedance equation. The results are the same as those calculated directly.

$$Z_{eq} = \frac{V_{TH}}{I_{SC}} = \frac{\left(\dfrac{Z_2}{Z_1 + Z_2} v\right)}{\dfrac{V_S}{Z_1}}$$

$$= \frac{Z_1 Z_2}{Z_1 + Z_2}$$

If any two of the parameters are known, then the other can be found.

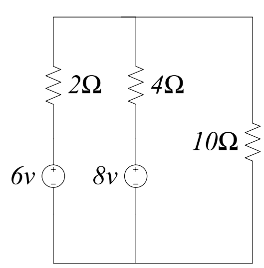
5.6.1 Example



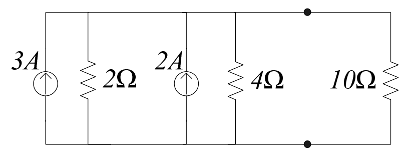
Find the equivalent circuit for the figure employed with other analysis methods.

Example

Redraw the circuit with the two voltage branches in parallel.



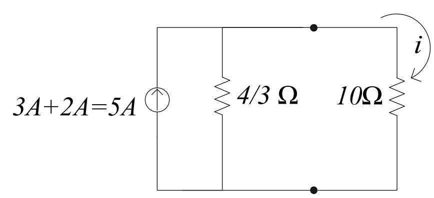
The voltage source in series with the impedance is a Thevenin source. The circuit can be converted into a Norton souce with a current source in parallel with the same impedance.



Simplify the circuit. Add the current sources at the node. Calculate the parallel impedance of the sources.

$$R_{TH} = 2 // 4$$
$$= \frac{2*4}{2+4} = \frac{4}{3}\Omega$$

Redraw the Norton equivalent source across the terminals.



The Norton equivalent source can be used to find the current across the terminals. This is a current divider through the 10Ω impedance.

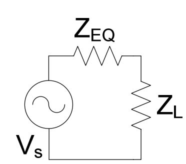
$$I = 5A\frac{\frac{4}{3}}{\frac{4}{3}+10} = 0.588A$$

Notice a variety of methods have been used to solve the same network. Use the method that is most convenient for the problem at hand.

5.7 Maximum power transfer

Energy and energy rate or power are the common basis for transferring between electrical, magnetic, and mechanical systems. Every energy conversion is less than 100% efficient. Therefore, there are losses that create heat.

Resistance is the element of conversion to mechanical energy. The load can be considered to be in series with a Thevenin equivalent source and impedance.



To obtain the maximum real power transfer involves matching the load impedance to the network equivalent impedance.

Real power is the product of voltage and current in phase that is projected onto the real axis at an angle of zero.

$$S = P = VI\cos\theta$$
$$Z = R = \frac{V}{I}$$
$$P = I^2 R_L$$

Real power is the current squared on the resistance. The current from the Thevenin circuit depends on the equivalent resistance and the load resistance.

$$I = \frac{V_S}{R_{TH} + R_L}$$

Then the power to the load resistance is arranged in terms of the voltage

$$P = \left(\frac{V_S}{R_{TH} + R_L} \right)^2 R_L$$

Maximizing power can be found by finding the derivative of power with respect to the load resistance, then setting the derivative to zero. The load resistance can then be verified.

$$P = V_S{}^2 \left[\frac{R_L}{\left(R_{TH} + R_L\right)^2} \right]$$

Take the derivative.

$$\frac{dP}{dR_L} = V_S{}^2 \left[\frac{\left(R_{TH} + R_L\right)^2 - R_L \times 2\left(R_{TH} + R_L\right)}{\left(R_{TH} + R_L\right)^4} \right]$$

$
universal engineering symbol

$, t, quality
engineering trade-offs

Maximize by setting the derivative to zero.

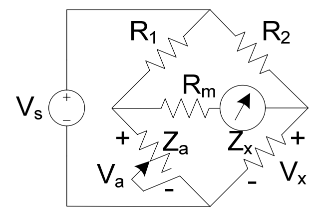
$$\left(R_{TH} + R_L\right)^2 = 2R_L \left(R_{TH} + R_L\right)$$
$$R_{TH} = R_L$$

Maximum power transfer occurs when the load resistance is equal to the source resistance.

	EXAMPLES
Ex 1.3-1	Situation: An audio amplifier of 5V is connected through an equivalent resistance of 8 Ohms. What is: total power and power delivered to an 8 Ohm speaker? $P = \frac{V^2}{\Sigma R} = \frac{5^2}{16} = \frac{25}{16} W$ $P_L = \frac{R_L}{\Sigma R} P = \left(\frac{8}{16}\right)\frac{25}{16} = \frac{25}{32}$
Ex 1.3-2	Situation: A mis-matched 4Ohm speaker is connected. What is: total power and power delivered to 4 Ohm speaker? $P = \frac{V^2}{\Sigma R} = \frac{5^2}{12} = \frac{25}{12} W$ $P_L = \frac{R_L}{\Sigma R} P = \left(\frac{4}{12}\right)\frac{25}{12} = \frac{25}{36}$

Ex 1.3-3	Situation: That did not work so well. A matching transformer is used that is rated at 16Ohms. What is: total power and power delivered to 16 Ohm speaker? $P=\frac{V^2}{\Sigma R}=\frac{5^2}{24}=\frac{25}{24}W$ $P_L=\frac{R_L}{\Sigma R}P=\left(\frac{16}{24}\right)\frac{25}{24}=\frac{25}{36}$
Ex 1.3-4	What is: comparison between halving and doubling the load resistance? Power delivered is the same and less than a matched load.

5.8 Wheatstone bridge



A Wheatstone bridge is used to measure the value of an unknown impedance very precisely. An unknown impedance is connected to the bridge. The standard or variable impedance is adjusted until the meter reads zero, indicating no current is flowing. Generally, the variable impedance is a box of very precise switched impedances. The standard inductor, capacitor, or resistors can be known with great precision.

Resistance measurement can be completed with a dc power source and a galvanometer. A galvanometer is essentially a D'Arsonval instrument operating as a sensitive current indicating meter. Reactance measurements are completed with a frequency source and alternating current meter.

No current flow through the meter implies three things. The impedances Z_1, and Z_a are operating in series. Similarly, Z_2, and Z_x are in series. Furthermore, V_a is equal to V_x.

Then a voltage divider can be used.

$$V_a=\frac{Z_a}{Z_1+Z_a}V_S$$

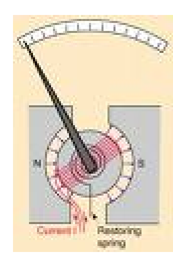
$$V_x=\frac{Z_x}{Z_2+Z_x}V_S$$

$$V_a=V_x$$

Consider balanced conditions.

$$\frac{Z_a}{Z_1+Z_a}=\frac{Z_x}{Z_2+Z_x}$$

$$\boxed{Z_x=\frac{Z_2}{Z_1}Z_a}$$



Note the impedance is a complex value that can be a series or parallel combination.

$$Z_x=R+jX$$

The unknown impedance and the standard can be inductors or capacitors to make a reactance. They can also be resistors. The other two impedances are generally resistance. Interestingly, the frequency of the source does not enter the calculation. Frequency can be divided from both sides of the equation. However, a frequency generator must be used to excite the storage devices.

	EXAMPLES
Ex 1.3-1	Situation: R1=2K, R2=3K, f=2kHz, Ca=10pf, 5 Vac. What is: unknown capacitance? $Z_x = \frac{Z_2}{Z_1} Z_a$ $\frac{1}{C_x} = \frac{3K}{2K}\frac{1}{10\,pf}$ $C_x = 6.67\,pf$

5.9 Problems

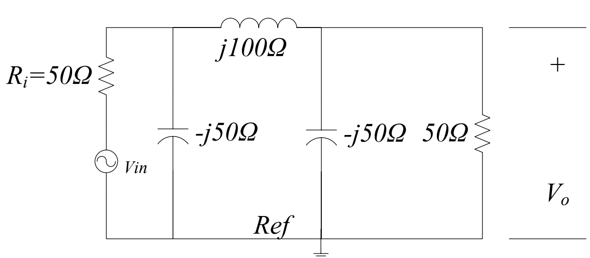
<u>Practice Problem</u>

SITUATION:
The circuit shown in the figure below is the pi representation of a transmission line.

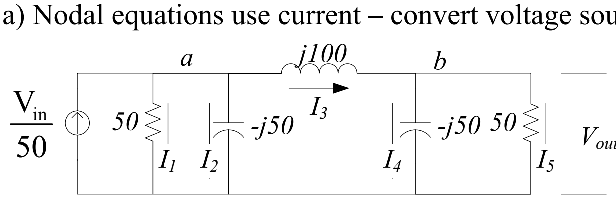
REQUIREMENTS:

Write a set of nodal equations for the circuit the "Ref" as the reference or common terminal.

Solve the equations of requirement (a) above for Vo if $V_{in} = 100\angle 0°$



SOLUTION:
Simple circuits problem
a) Nodal equations use current – convert voltage sources to current sources.



$$I_1 = \frac{v_a - 0}{50} \qquad I_2 = \frac{v_a - 0}{-j50} \qquad I_3 = \frac{v_a - v_b}{j100} \qquad I_4 = \frac{v_b - 0}{-j50} \qquad I_5 = \frac{v_b - 0}{50}$$

KCL @ a

$$\frac{V_{in}}{50} = I_1 + I_2 + I_3$$

$$\frac{V_{in}}{50} = \frac{v_a}{50} + \frac{v_a}{-j50} + \frac{v_a}{j100} - \frac{v_b}{j100}$$

$\therefore$

$$(2 + j1)v_a + j1v_b = 2V_{in}$$

KCL @ b

$$I_3 - I_4 - I_5 = 0$$

$$\frac{v_a}{j100} - \frac{v_b}{j100} - \frac{v_b}{-j50} - \frac{v_b}{50} = 0$$

$\therefore$

$$j1v_a + (2 + j1)v_b = 0$$

$$v_b = v_{out}$$

Solve using Cramer's Rule – Substitute RHS for v_b

$$v_{out} = \frac{\begin{vmatrix} 2 + j1 & 2V_{in} \\ j1 & 0 \end{vmatrix}}{\begin{vmatrix} 2 + j1 & j1 \\ j1 & 2 + j1 \end{vmatrix}} = \frac{-j2V_{in}}{4 + j4 - 1 + 1} = \frac{2V_{in}\angle -90}{4\sqrt{2}\angle 45}$$

$$v_{out} = 0.3535\angle -135 * V_{in}$$

b) For $V_{in}=100\angle 0$

$$v_{out} = 100\angle 0 * 0.3535\angle -135 = 35.35\angle -135$$